

Recursion Schemes

folds (tear down a structure)

$algebra\ f\ a \rightarrow Fix\ f \rightarrow a$

unfolds (build up a structure)

$coalgebra\ f\ a \rightarrow a \rightarrow Fix\ f$



generalized $(f\ w \rightarrow w\ f) \rightarrow (f\ (w\ a) \rightarrow \beta)$	catamorphism $f\ a \rightarrow a$	anamorphism $a \rightarrow f\ a$	generalized $(m\ f \rightarrow f\ m) \rightarrow (a \rightarrow f\ (m\ \beta))$
	prepromorphism* ... after applying a NatTrans $(f\ a \rightarrow a) \rightarrow (f \rightarrow f)$	postpromorphism* ... before applying a NatTrans $(a \rightarrow f\ a) \rightarrow (f \rightarrow f)$	
	paramorphism* ... with primitive recursion $f\ (Fix\ f\ \times\ a) \rightarrow a$	apomorphism* ... returning a branch or single level $a \rightarrow f\ (Fix\ f\ \vee\ a)$	
	zygomorphism* ... with a helper function $(f\ b \rightarrow b) \rightarrow (f\ (b\ \times\ a) \rightarrow a)$	g apomorphism $(b \rightarrow f\ b) \rightarrow (a \rightarrow f\ (b\ \vee\ a))$	
g histomorphism $(f\ h \rightarrow h\ f) \rightarrow (f\ (w\ a) \rightarrow a)$	histomorphism ... with prev. answers it has given $f\ (Cofree\ f\ a) \rightarrow a$	futumorphism ... multiple levels at a time $a \rightarrow f\ (Free\ f\ a)$	g futumorphism $(h\ f \rightarrow f\ h) \rightarrow (a \rightarrow f\ (m\ a))$

refolds (build up then tear down a structure)

$algebra\ g\ b \rightarrow (f \rightarrow g) \rightarrow coalgebra\ f\ a \rightarrow a \rightarrow b$

hylomorphism

cata; ana

dynamorphism

histo; ana

codynamorphism

cata; futu

chronomorphism

histo; futu

Elgot algebra

... may short-circuit while building
 $cata; a \rightarrow b \vee f\ a$

coElgot algebra

... may short-circuit while tearing
 $a \times g\ b \rightarrow b; ana$

reunfolds (tear down then build up a structure)

$coalgebra\ g\ b \rightarrow (a \rightarrow b) \rightarrow algebra\ f\ a \rightarrow Fix\ f \rightarrow Fix\ g$

metamorphism

ana; cata

generalized

apply ... both ... [un]fold

others

synchronorphism

???

exomorphism

???

mutumorphism

... can refer to each other's results

$(f\ (a \times b) \rightarrow a) \rightarrow (f\ (a \times b) \rightarrow b)$

generalized

apply the generalizations for both
the relevant fold and unfold

Stolen from Edward Kmett's <http://comonad.com/reader/2009/recursion-schemes/>

* This gives rise to a family of related recursion schemes, modeled in recursion-schemes with distributive law combinators

These can be combined in various ways. For example, a “zygohistomorphic prepromorphism” combines the zygo, histo, and prepro aspects into a signature like $(f\ b \rightarrow b) \rightarrow (f \rightarrow f) \rightarrow (f\ (w\ (b\ \times\ a)) \rightarrow a) \rightarrow Fix\ f \rightarrow a$